Shunt Impedance of Klystron Cavities

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Summary—**Values of R/Qf or re-entrant resonators are given for configurations for which neither the coaxial-field nor the radial-field approximations are valid. The values are calculated by net point methods, and agree well with experimental values obtained by perturbation methods. Some errors arising from the finite dimensions of perturbing plungers are also discussed.**

General

CYLINDRICAL re-entrant cavities operating in their principal mode of oscillations comprise a large class of cavities suitable for klystron operation. Since the exact solutions of Maxwell's equations inside cavities of these configurations are not known, the properties of interest (such as resonant frequency, shunt impedance, and Q) cannot be calculated exactly, but must ordinarily be determined by experiment. Two limiting cases for which the fields are known[1] analytically are those of a coaxial line ($z_0 \gg \rho_2 - \rho_1$) and those of a radial line ($z_0 \ll \rho_2 - \rho_1$), for which calculations of cavity properties can be made (see Fig. 1). The accuracy of these calculations is limited by how well one can approximate the fringing fields near the gap.[2]

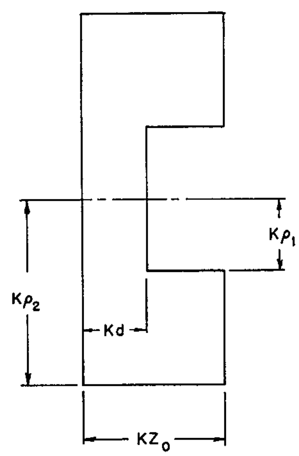
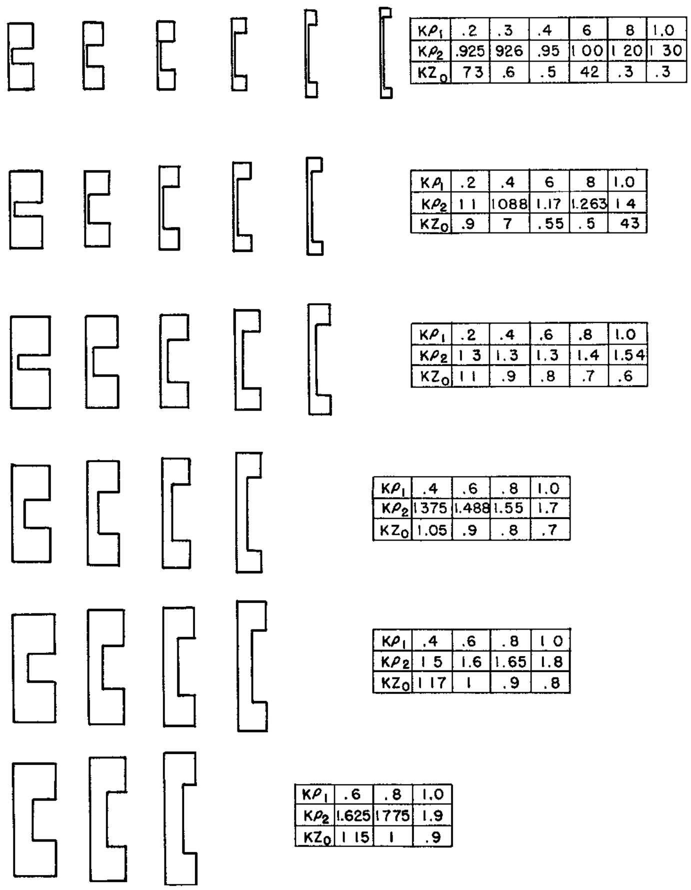


Fig. 1—Typical reentrant cavity.

The resonant frequencies of these types of cavities have been estimated to good accuracy by assuming reasonable fields inside the cavities. The results are given adequately in available design charts.[3] It is fortunate that the value of the resonant frequency is relatively independent of the choice of fields. The Q and the shunt impedance for some configurations, using the same approximate method, have been given.[4] This reference is not generally available; moreover, the accuracy of the results is limited. A more accurate approach is due to Fenichel,[5] who obtained the fields inside the resonator using net point methods. The accuracy obtainable is limited only by how fine a mesh of points one is willing to consider. The configurations selected by Fenichel (Fig. 2) were those which fall in the class of "square"



$K\rho_1$	.2	.3	.4	6	8	1.0
$K\rho_2$	.925	926	.95	1 00	1 20	1 30
KZ_0	73	.6	.5	42	.3	.3

$K\rho_1$	.2	.4	6	8	1.0
$K\rho_2$	1 1	1088	1.17	1.263	1 4
KZ_0	.9	7	.55	.5	43

$K\rho_1$	.2	.4	.6	.8	1.0
$K\rho_2$	1 3	1.3	1.3	1.4	1.54
KZ_0	1 1	.9	.8	.7	.6

$K\rho_1$	.4	.6	.8	1.0
$K\rho_2$	1375	1.488	1.55	1.7
KZ_0	1.05	.9	.8	.7

$K\rho_1$	.4	.6	.8	1 0
$K\rho_2$	1 5	1.6	1.65	1.8
KZ_0	1 17	1	.9	.8

$K\rho_1$	.6	.8	1.0
$K\rho_2$	1.625	1775	1.9
KZ_0	1 15	1	.9

Fig. 2—Normalized dimensions of investigated cavities.

resonators ($z_0 = \rho_2 - \rho_1$), where neither the coaxial nor the radial approximation is valid. Values of R/Q and QS/λ are shown in Figs. 3 and 4, respectively. The initial estimates of the field were obtained by analog methods developed at Stanford University.[6]

The purpose of this report is to ascertain to what extent one can rely on net point calculations in the range of configuration for which neither the radial nor the coaxial approximation is valid, as well as to evaluate some of the experimental errors in the measurement of the properties of such cavities.

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[1] M. Ettenberg, "Calculation of Q and shunt impedance of re-entrant cavities," Sperry Gyroscope Co. Rep. No. 5221-1076; January 3, 1947.

[2] J. R. Whinnery, "Radial-line discontinuities," Proc. IRE, vol. 43, pp. 46–51; January, 1955.

[3] See, e.g., T. Moreno, "Microwave Transmission Design Data," McGraw-Hill Co., New York, N. Y., pp. 230–238; 1948.

[4] "Microwave Transmission Design Data," Publication No. 23-80, Sperry Gyroscope Co., pp. 206–208; 1944.

[5] L. Fenichel, "Net point calculations for fields in reentrant cavities," M.S. Thesis, Stanford University; May, 1948.

[6] F. W. Schott, "A network analogue approach to the study of cavity resonator losses," TR-No. 4 (N6onr 25107), Stanford University (Electronics Research Laboratory); June 30, 1948.

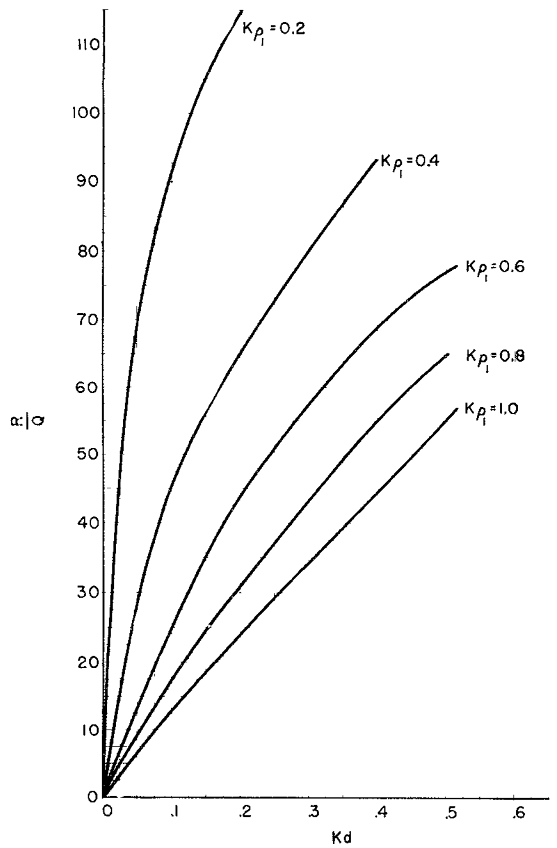

Fig. 3—R/Q of "square" resonators by Net Point methods.

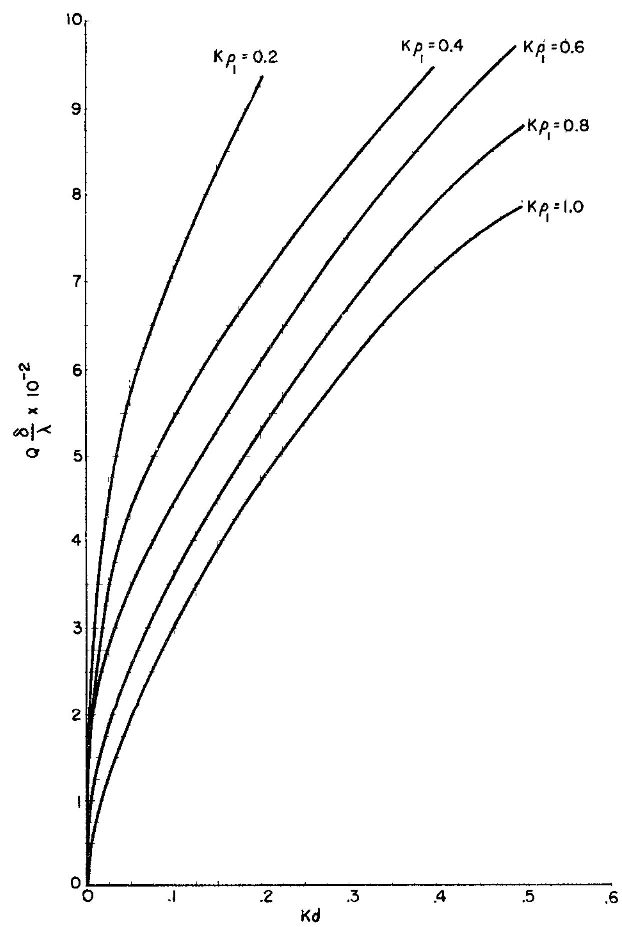

Fig 4—$Q\delta/\lambda$ of "square" resonators by Net Point methods.

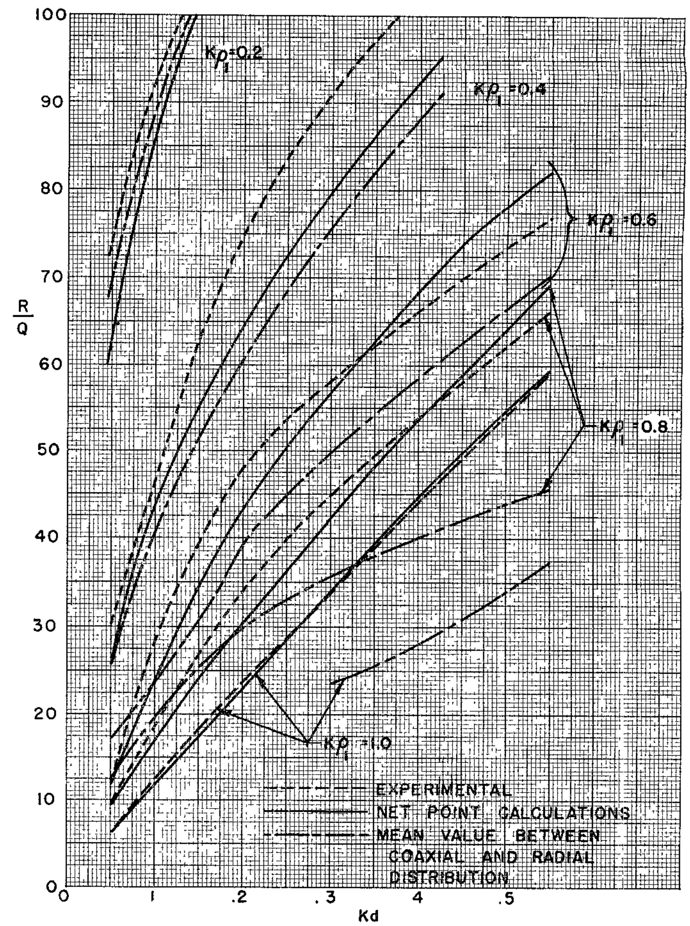

Fig. 5—Comparison of experimental and calculated values of R/Q.

Experimental Results

The experimental confirmation of these results consisted of measuring the Q of the cavities by standard Q-circle methods; measuring the shunt impedance by inserting calibrated high-resistance liquid columns across the gap of the cavity and comparing Q's before and after insertion of column; and measuring R/Q of these cavities by perturbation techniques.[7] Although agreement obtained in the measurement of Q and shunt impedance was encouraging, it was not of sufficient accuracy to be of real value. R/Q measurements, independent of cavity loss, can be performed with good accuracy. Fig. 5 indicates results of these measurements for "square" resonators. Three sets of curves are shown:

(a) calculated curves using net point methods;

(b) calculated curves based on a mean value between co-axial and radial field approximation; and

(c) measured values, as determined by R/Q perturbation experiments.

Agreement between net point calculations and experimental curves is very good, better than 4 per cent in most cases. The calculation of mean values between coaxial and radial distributions show fair agreement in regions of small gap spacings and large posts, but are lower than experimental values in regions of large gaps

[7] W. W. Hansen and R. F. Post, "On the measurement of cavity impedance," *J. Appl. Phys.*, vol. 19, pp. 1059–1061; November, 1948.

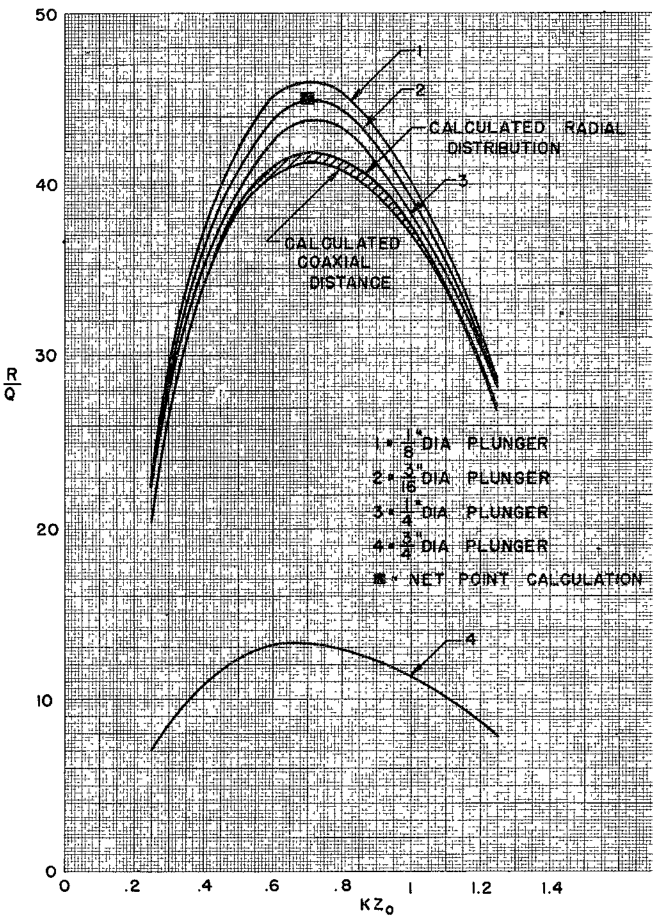

Fig. 6—R/Q as a function of perturbing plunger size for cavities of same gapconfi guration, varying from purely radial to purley coaxial limit.

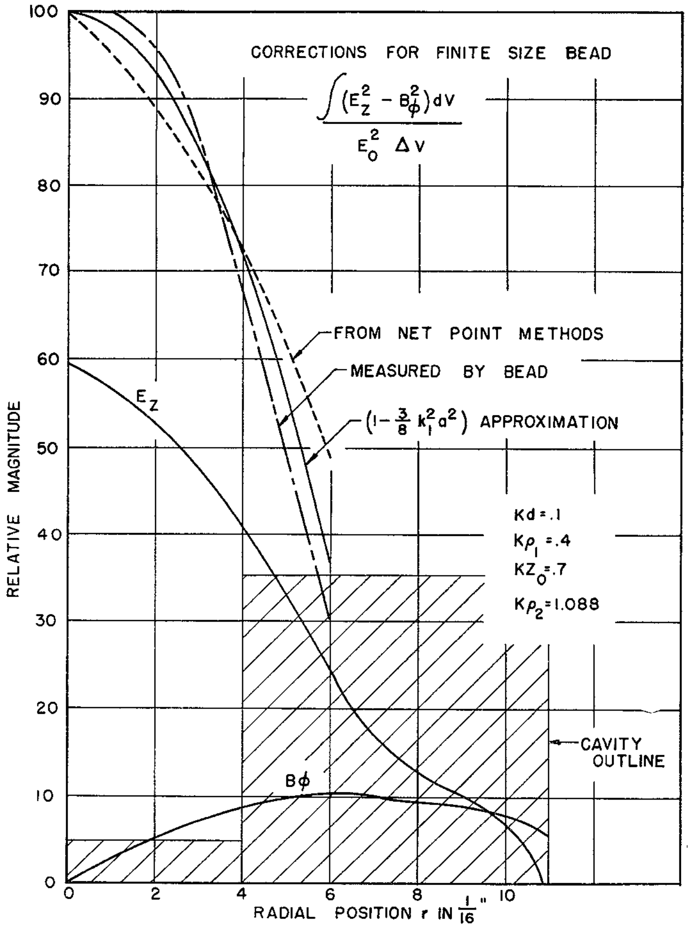

Fig. 7—Correction for finite size plunger.

and small posts. When reasonable assumptions regarding fringing fields in the gap are made, closer agreement should be obtained.

The conclusions to be drawn from these curves indicate that for resonators in which neither approximation is valid, the mean values between calculated radial and coaxial distributions are reasonably accurate (not more than 15 per cent too low) for values of $k\rho_1$ up to 0.6 provided that $kd<0.5$, and for values of $k\rho_1$ from 0.6 to 0.8 provided that $kd<0.3$. For larger values of $k\rho_1$, the accuracy becomes extremely poor. In these measurements, the size of the perturbing plunger was sufficiently small so that the integral $\int(\epsilon E^2-\mu H^2)dV$ over the perturbing volume could be accurately approximated by $E_0^2\Delta V$, where E_0 is the field of interest in evaluating the shunt impedance.[8]

In order to explore possible generalizations of these conclusions, a set of cavities was built, all having the same post diameter and gap spacing ($kd=0.1$, $k\rho_1=0.4$), resonant at 10 cm, and ranging from an almost purely radial cavity to an almost purely coaxial cavity. The results of R/Q measurements on these cavities are given in Fig. 6, for various plunger sizes. The results again indicate that assumed fields predict values low by not more than 12 per cent and indicate much better agreement for cavities which are not "square."

[8] *Ibid.*

Some Experimental Errors in the Measurements of R/Q

The measurement of R/Q is relatively simple and can be carried out rapidly without undue difficulty. A small perturbation of volume ΔV is inserted in the gap region of the cavity, and the change in resonant wavelength $\Delta\lambda$ is accurately measured. The experiment is repeated, with different volumes of perturbation, and a curve of $\Delta\lambda$ vs ΔV is constructed. Although the method is easily extended to gridless cavities, for convenience we shall confine our discussion to gridded cavities. The relation for R/Q is given by

$$\frac{R}{Q} = \frac{120d^2}{1-\alpha} \lim_{\Delta V\to 0} \frac{\Delta\lambda}{\Delta V} \quad (1)$$

where $\Delta\lambda$ is the change in resonant wavelength for a given volume of perturbation ΔV, d is the gap spacing, and α is a correction factor described below.

The constant is correct for the case of perturbing button shapes which do not affect fields far away, such as a cylinder whose volume is made to shrink to zero by reducing its height. For large buttons, the E field may not be constant over the plunger area, and the B field may not be zero. In this case, (1) must be corrected, since the plunger measures the integral $\int(\epsilon E^2-\mu H^2)dV$ rather than the desired $E_0{}^2\Delta V$. This can be taken into account by the correction factor α which is approximately given by $\alpha=(\frac{3}{8})\ (ka)^2$, assuming the fields vary in a manner similar to the principal mode in a cylindrical cavity. Here a is the plunger radius and $k_1=2.4/\rho_2$. Results for a particular case are shown in Fig. 7 (previous page), indicating that the correction is fair for the particular configuration. The accuracy of this correction for other configurations is a matter of some conjecture. In general, for the best results, the perturbing volume should be as small as possible.

The effect of coupling-loop size and contact pressure on R/Q measurements was also investigated, and as expected, did not affect the value of R/Q, but shifted the resonant frequency slightly.

Conclusions

The values of R/Q of klystron cavities obtained by using radial-field or coaxial-field distributions are very useful, except for a class of configurations approaching "square" resonators. For this class of cavities, the mean value of the two field approximations gives values of R/Q which are in general too low. The error, as determined by experiment and net point calculations, does not exceed 15 per cent, for values of $k\rho_1 \leqq 0.6$ (provided $kd<0.5$) and for values of $k\rho_1 \leqq 0.8$ (provided $kd<0.3$). Accurate values of R/Q for these configurations are also given. For cavities which do not fall within above specifications, the fields should be determined by net point methods, or by experiment. Effects of perturbing plunger sizes on R/Q measurements may also be important if plunger diameter is comparable to post diameter; a correction factor should then be applied.

Planar Transmission Lines—II*

DAVID PARK†

Summary—**An expression is found for the characteristic impedance of a transmission line consisting of two parallel strips of foil placed between, and perpendicular to, two wide plates.**

Introduction

CONTINUING the investigation of an earlier paper,[1] of transmission lines composed of flat strips of metal or foil, we examine here the characteristics of a line in which the strips no longer lie in the same plane. We shall be concerned with a configuration, shown in Fig. 1(a), in which the two center strips are perpendicular to the top and bottom sheets, midway between them, and separated from each other by distance $2D$. The center strips are each of height $2C$, and the separation between the top and bottom sheets is $2H$. The center strips are driven, and the top and bottom sheets are considered to be electrically neutral and effectively infinite in width.[2] We shall use the notations and, as far as possible, the results of the earlier paper to find, by the method of conformal mapping, the characteristic impedance Z_0 of the line in terms of H, C, D, and the dielectric constant κ of the dielectric material between the plates. (To calculate the attenuation by the methods of the earlier paper is straightforward though rather onerous, and we have not carried it out.)

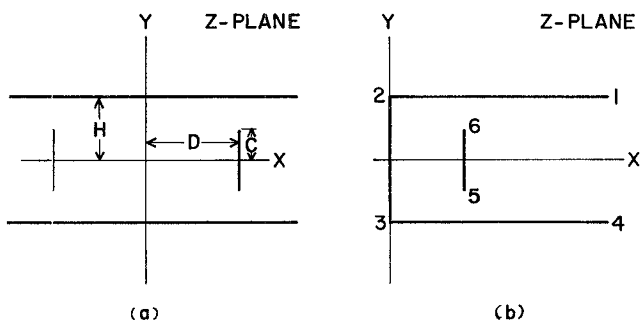

Fig. 1—(a) Cross section of the transmission line. The vertical strips are driven and the horizontal plates, assumed very wide, are electrically neutral. (b) Arrangement electrically equivalent to (a). Some significant points are numbered.

General Formalism

To begin with, let us substitute for the arrangement of Fig. 1(a) that of Fig. 1(b), in which the left-hand side of the line is substituted by its image in the vertical center plane. As mentioned,[3] the line 1234 can be mapped by the transformation

$$z = A \sinh Z/B \tag{1}$$

into the y axis of the z plane, if

$$(1/2)\pi B = H. \tag{2}$$

We must now see what this mapping does to the internal

* Supported by Sprague Electric Co., North Adams, Mass.

† Williams College, Williamstown, Mass. Fullbright Lecture in Physics at the Univ. of Ceylon, 1955–6).

[1] D. Park, "Planar transmission lines," Trans. IRE, vol. MTT-3, pp. 8–12; April, 1955.

[2] *Ibid.*, section 5, for an examination of this assumption in the case dissussed there.

[3] *Ibid.*